Here is an article for all of the functions and graphs used in Maths Methods Units 1 & 2. You can use these as study notes if you like. To look for any diagrams on the shape of graphs, refer to your textbook.

Linear Graphs

- The form is $y = mx + c$, where m is the gradient and c is the y-intercept.
- The gradient between two points is equal to $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$
- Two lines with the same gradient are parallel to each other and do not intersect.
- To find the equation of a straight line given two points, use the formula:
  $m(x - x_1) = y - y_1$, where m is the gradient.
- When given equations of the form $ax + by + c = 0$, then transpose to $y = mx + c$
- Vertical lines are of the form $x = a$, where the x-axis intercept is given as $(a, 0)$.
- Horizontal lines are of the form $y = c$, where the y-axis intercept is given as $(0, c)$.
- Perpendicular lines can be calculated using the formula: $m_1 m_2 = -1$.
- To calculate the tangent of the angle of slope, calculate the gradient, then substitute it into tan □ to get the angle.
- The distance between two points is given as $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- The midpoint of a line segment is given as $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Parabolas

- The form is $y = ax^2 + bx + c$, which is known as polynomial form.
- The form is $y = a(x - b)^2 + c$, which is known as turning point form.
- Parabolas of the form $y = ax^2$, in which the axis of symmetry is $x = 0$, and the turning point is (0,0). If a is greater than 1, the graph will be narrower than $y = x^2$. If it is between 0 and 1, the graph will be wider. If a is negative, then the graph is inverted or reflected in the x-axis.
- Parabolas of the form $y = ax^2 + c$ involve a translation up or down the y-axis. The axis of symmetry is $x = 0$ and the turning point is $(0, c)$, which correlates to the y-intercept.
- Parabolas of the form $y = a(x - b)^2$ involve a translation left or right along the x-axis. The axis of symmetry is $x = -b$ and the turning point is $(-b, 0)$, which correlates to the x-intercept.
- The turning point of a parabola is $(-b, c)$ from the turning point form.
- The turning point of a parabola from polynomial form can be found by completing the square so it is in turning point form.
- To find x-intercepts, either factorise the equation or use the quadratic formula:
  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$,
- The axis of symmetry is $x = -b$ from the turning point form.
- The axis of symmetry is $x = \frac{-b}{2a}$ from the polynomial form.
- The discriminant is equal to $\Delta = b^2 - 4ac$.

- If Δ < 0, then there are no real solutions.
- If Δ = 0, then there is one rational solution.
- If Δ > 0, then there are two solutions. If it results in a perfect square, there are two rational solutions. If it is not a perfect square, there are two irrational solutions.

Rectangular Hyperbolas

- The form is $y = a / (x - h) + k$.
- If a is greater than 1, then the graph is dilated from the x-axis, or moves further away from the x-axis, compared to the graph of $y = 1 / x$. If a is between 0 and 1, then the graph is dilated to the x-axis, or moves closer to the x-axis. If a is negative, then the graph is inverted or reflected in the y-axis.
- The asymptotes are given as $x = -h$ and $y = k$.
- The x-intercepts can be calculated by substituting $y = 0$.
- The y-intercepts can be calculated by substituting $x = 0$.

The Truncus

- The form is $y = a / (x – h)^2 + k$.
- If a is greater than 1, then the graph is dilated from the x-axis, or moves further away from the x-axis, compared to the graph of $y = 1 / x^2$. If a is between 0 and 1, then the graph is dilated to the x-axis, or moves closer to the x-axis. If a is negative, then the graph is inverted or reflected in the x-axis.
- The asymptotes are given as $x = -h$ and $y = k$.
- The x-intercepts can be calculated by substituting $y = 0$.
- The y-intercepts can be calculated by substituting $x = 0$.

The Square Root Function

- The form is $y = a\sqrt{-(x – h)} + k$
- If a is positive, then the top half of the graph will be sketched. If a is negative, then the bottom half of the graph will be sketched.
- The stationary point, or turning point, is determined by the coordinates ($-h$, k).
- If the expression under the radical is negative, then the graph is reflected in the y-axis.
- If the entire equation is negative, then the graph is reflected in the x-axis.
- The x-intercepts can be calculated by substituting $y = 0$.
- The y-intercepts can be calculated by substituting $x = 0$.

Circles and Semicircles

- The form is $x^2 + y^2 = r^2$ for circles with a centre at (0,0).
- The form is $(x – h)^2 + (y – k)^2 = r^2$ for circles not centred at the origin. This is equal to the form $x^2 + y^2 – 2hx – 2ky + c = 0$, where $c = h^2 + k^2 – r^2$.
- The centre of the circle is denoted as ($-h$,$-k$).
- The radius of the circle is equal to r.
- Circles of the form $x^2 + y^2 – 2hx – 2ky + c = 0$ can be sketched by completing the square, so it is in the form $(x – h)^2 + (y – k)^2 = r^2$.

- When sketching semicircles, draw the graph of the circle. If there is a positive sign in front of the radical, then draw the top half of the circle. If there is a negative sign, then draw the bottom half of the circle.
- The x-intercepts can be calculated by substituting $y = 0$.
- The y-intercepts can be calculated by substituting $x = 0$.

Cubic Graphs

- The form is $y = ax^3 + bx^2 + cx + d$, which is in polynomial or factor form.
- The form is $y = a(x - h)^3 + k$, which is the basic form.
- The point of inflexion from the basic form is given as $(-h, k)$. This only goes for graphs that resemble this form.
- Cubic graphs are said to take two distinct patterns: the basic form and the factor form, plus their negative forms.
- To find the x-intercepts, divide the polynomial so it is in factor form. Then use the Null Factor Law to find the x-intercepts.
- If there is a repeated factor in the equation, then the graph will only have two x-intercepts.
- If, when factorised, one linear factor is produced, then there is one x-intercept. This is because the other factors cannot be simplified another further.
- Before sketching the graph, draw a sign diagram that helps you to sketch the graph more accurately.
- The y-intercept can be found by substituting $x = 0$.

Quartic Graphs

- The form is $y = ax^4 + bx^3 + cx^2 + dx + e$, which is in polynomial or factor form.
- The form is $y = a(x - h)^4 + k$, which is the basic form.
- The turning point from the basic form is given as $(-h, k)$. This only goes for graphs that resemble this form.
- Quartic graphs are said to take 6 distinct patterns, which are:
  - $y = ax^4$
  - $y = ax^4 + cx^2$
  - $y = ax^2(x - b)(x - c)$
  - $y = a(x - b)^2(x - c)^2$
  - $y = a(x - b)(x - c)^3$
  - $y = a(x - b)(x - c)(x - d)(x - e)$
- To find the x-intercepts, divide the polynomial so it is in factor form. Then use the Null Factor Law to find the x-intercepts.
- When factorised, the number of factors is the same as the number of x-intercepts.
- Before sketching the graph, draw a sign diagram that helps you to sketch the graph more accurately.
- The y-intercept can be found by substituting $x = 0$.

Exponential and Logarithmic Functions

- The form is $y = k \times a^{bx} + c$.
- For simple graphs of the form $y = a^x$, the only asymptote is $y = 0$ or the x-axis. The y-intercept will always be equal to 1.

- If the value of a is greater than 1, the graph has a positive gradient. This means that when x approaches ∞, then y will increase exponentially. If the value of a is between 0 and 1, then this will produce a negative index. Therefore, this is the same as the graph with the positive index, but is reflected in the y-axis.
- For graphs of the form $y = k \times a^{bx} + c$, the following apply:
  - The horizontal asymptote is equal to $y = c$.
  - The y-intercept is equal to $(0, k + c)$
  - If b is a number other than 1, then every x value must be divided by b to get the points of the transformed graph.
  - If k is a number other than 1, then every y value must be multiplied by k to get the points of the transformed graph.
  - If k is negative, then the graph of its positive form is reflected in the horizontal asymptote.
- Logarithmic functions are the inverse of exponential functions. This means that they are exponential functions reflected in the line $y = x$.
- For simple graphs of the form $y = \log_a x$, the only asymptote is $x = 0$ or the y-axis. The x-intercept will always be equal to 1.
- If the value of a is greater than 1, the graph has a positive gradient. This means that when x approaches ∞, then y will increase logarithmically. If the value of a is between 0 and 1, then this will produce a negative logarithm. Therefore, this is the same as the graph with the positive index, but is reflected in the x-axis.
- For graphs of the form $y = \log_a(x - c)$, the following apply:
  - The vertical asymptote is equal to $x = c$.
  - The x-intercept is equal to $(1 + c, 0)$.

<u>Sine and Cosine Functions</u>

- The form is $y = a \sin n(x \pm b) + c$ or $y = a \cos n(x \pm b) + c$.
- The basic graph is $y = \sin x$. It has the following characteristics:
  - The graph has a wavelength or period of 2π units. It is a periodic function.
  - The maximum and minimum values are $y = 1$ and $y = -1$. This means that the amplitude is equal to 1.
  - The y-intercept is equal to $(0, 0)$.
- The other basic graph is $y = \cos x$. It has the following characteristics:
  - The graph has a wavelength or period of 2π units. It is a periodic function.
  - The maximum and minimum values are $y = 1$ and $y = -1$. This means that the amplitude is equal to 1.
  - The y-intercept is equal to $(0, 1)$.
- In the graphs of the form $y = a \sin(nx)$ and $y = a \cos(nx)$, the following characteristics apply:
  - The graph has a wavelength or period of $2\pi/n$.
  - The amplitude is always equal to a.
  - The maximal domain is equal to R.
  - The range of each function is $[-a, a]$.
  - If a is negative, then the graph of the positive form is reflected in the x-axis.
- In graphs of the form $y = a \sin n(x \pm b)$ and $y = a \cos n(x \pm b)$, a horizontal translation along the t-axis occurs, with the graph moving b units to the left or right.

- In graphs of the form $y = a \sin n (x \pm b) + c$ and $y = a \cos n (x \pm b) + c$, a horizontal and vertical translation has occurred. The graph is moved b units to the left or right, then moved c units up or down.

Tangent Function

- The form is $y = k \tan n (x \pm b) + c$.
- The basic graph is $y = \tan x$. It has the following characteristics:
  - The graph has a period of π units.
  - The x-intercepts are equal to $x = k\pi$.
  - The range is equal to R.
  - The equations of the asymptotes are equal to $y = (2k + 1) \pi / 2$.
- The graphs of the form $y = a \tan (nx)$, the following characteristics apply:
  - The period of the function is π / n.
  - The range of the function is R.
  - The asymptotes have equations $y = (2k + 1) \pi / 2n$.
  - The x-axis intercepts are equal to $x = k\pi / n$.
- The graphs of the form $y = a \tan (nx)$ involve:
  - A dilation of factor a from the x-axis.
  - A dilation of factor $1 / n$ from the y-axis.
- In graphs of the form $y = k \tan n (x \pm b)$, a horizontal translation along the x-axis occurs, with the graph moving b units to the left or right.
- In graphs of the form $y = k \sin n (x \pm b) + c$, a horizontal and vertical translation has occurred. The graph is moved b units to the left or right, then moved c units up or down.